The Sidereal Messenger.

CONDUCTED BY WM. W. PAYNE,

Director of Carleton College Observatory.

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Thou Lord in the beginning hast laid the foundation of the earth, and the heavens are the works of thy hands.

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ON THE REPRESENTATION OF COMETS' ORBITS BY MODELS.

WM. HARKNESS.

When a comet is discovered the first question asked about it is "What are its elements," and yet to the vast majority of amateurs these elements are almost unintelligible, and even to adepts they often convey but a vague idea of the true form and position of the orbit. The best way to realize their exact import is by making a model; and by showing how easily that can be done, it is hoped a fruitful source of instruction and amusement will be brought within reach of every one interested in the subject.

The orbits of all heavenly bodies are conic sections whose size, form, and position in space are defined by six quantities called elements, which, for brevity, are usually designated by the following symbols:

T = instant of the body's perihelion passage.

π = longitude of the perihelion; in the case of a comet, measured along the ecliptic from the vernal equinox to the comet's ascending node, and thence along the comet's orbit to its perihelion; in the case of the earth, measured along the ecliptic from the vernal equinox to the perihelion.

☊ = longitude of the ascending node; measured on the ecliptic, from the vernal equinox to the ascending node of the orbit.

i = inclination of the plane of the orbit to the plane of the ecliptic.

e = eccentricity of the orbit, sometimes given in parts of radius, sometimes in seconds of arc, and sometimes as an angle, φ. Parts of radius are most convenient for our purpose, and seconds of arc may be reduced to that unit by dividing them by 206,265″. When φ is given, $e = \sin \varphi$.

q = perihelion distance of the body; expressed in terms of the mean radius of the earth's orbit as unity.

For a parabolic orbit e is unity, and in that case the elements are frequently given by stating T, ω, ☊, i, and log. q. Here π has been replaced by

$$\omega = \pi - ☊ \qquad (1)$$

which is counted in the comet's orbit, backward, from the perihelion to the ascending node; and the perihelion will lie on the northern or southern side of the ecliptic according as w is less or greater than 180°.

As π and ☊ are counted from the vernal equinox, and i is measured from the plane of the ecliptic, these quantities necessarily refer to a particular equinox which is always specified.

It was long customary to measure longitudes in comets' orbits in the direction of the earth's motion, to limit i to the first quadrant, and to specify the direction of the comet's motion, whether direct or retrograde; but most astronomers now prefer to follow Gauss in regarding retrograde motion as a result of the inclination passing into the second quadrant, and in accordance with that view they measure a comet's longitude always in the direction of its own motion, and permit i to take any value between 0° and 180°. The circumstance that i is measured at the ascending node limits its range to the first and second quadrants, for if it were to pass into the third or fourth quadrant the ascending node would be converted into a descending one. For a comet having direct motion the numerical values of the elements are the same in the old system as in Gauss' system, but for a comet having retrograde motion they are different, and in that case, if their values according to the old system are designated by a sub-

script o, the equations requisite for passing from the old to the Gaussian system are

$$i = 180^\circ - i_0 \qquad \omega = 360^\circ - \omega_0 = -\omega_0$$

$$\Omega = \Omega_0 \qquad \pi = 2\Omega_0 - \pi_0$$

There is frequently much confusion respecting the angles π and ω, and as no model can be constructed without using the latter, it is important to have a clear understanding of its relations to π and Ω. In the old system of elements π is measured from the vernal equinox, along the ecliptic in the direction of the earth's motion, to the ascending node of the comet, and thence along the comet's orbit, *still in the direction of the earth's motion*, to the comet's perihelion. n Gauss' system π is measured from the ver ·nox, along the ecliptic in the direction of the earths's motion, to the ascending node of the comet, and thence along the comet's orbit, *in the direction of the comet's motion*, to the comet's perihelion. These definitions may perhaps be elucidated by the following statement: Imagine a perpendicular to the plane of the ecliptic, erected from the sun. Then to an observer situated north of the ecliptic in that perpendicular, the motion of the earth will be counter-clockwise, and longitudes in the earth's orbit will increase in that direction. Now consider a comet's orbit, imagine a perpendicular affixed to it in such a way that when the inclination of the orbit to the plane of the ecliptic is i the inclination of the perpendicular shall be $(i+90^\circ)$, and suppose an observer so situated in the perpendicular that when $i=0^\circ$ he shall be north of the ecliptic. Then, according to the old system of elements, for all possible values of i the observer will remain north of the ecliptic, and the motion of the comet will appear to him as counter-clockwise when direct, and clockwise when retrograde; but according to Gauss' system of elements, he will be north of the ecliptic when i is less than 90°, south of it when i is greater then 90°, and to him the apparent direction of the comet's motion will always be counter-clockwise. Whichever system is adopted, from his point of view π will always increase counter-clockwise, and to find the intersection of the plane of the

comet's orbit with the plane of the ecliptic, or in other words, the line of nodes, he must set off ω clockwise from the perihelion of the orbit.

In constructing a model of a comet's orbit two distinct processes are involved, namely: 1st, the drawing of the curves destined to represent the orbits of the earth and comet, and the marking upon them of the line of nodes, direction of motion of the bodies, etc., as they would be seen by an observer perched upon the imaginary perpendicular described in the preceding paragraph; and 2nd, the putting of these curves together.

In general, the curves must be drawn from their equations, the constants of the latter being expressed in terms of the elements. For a parabola

$$y^2 = 2px \tag{2}$$

and as the distance from the vertex of the curve to the focus is $\frac{1}{2}p$, for a parabolic orbit $\frac{1}{2}p=q$, and therefore

$$y^2 = 4qx \tag{3}$$

The easiest way of constructing this curve is by drawing it through a series of points laid down by means of their co-ordinates computed from equation (3).

For an ellipse

$$y = \frac{B}{A}\sqrt{(A^2-x^2)} \tag{4}$$

in which the values of the semi-major and semi-minor axes are respectively

$$A = \frac{q}{1-e} \tag{5}$$

$$B = A\sqrt{(1-e^2)} \tag{6}$$

The lengths of the axes having been computed and plotted, the foci can be found by construction, and then the curve can be very readily drawn by a pencil moving in the bight of a thread whose two extremities are attached to pins stuck into the foci.

For an ellipse of small eccentricity, a circle of radius $\frac{1}{2}(A+B)$ may often be substituted, and then the distance from the centre of the circle to the focus will be $\frac{1}{2}(A+B)e$.

For the earth's orbit, at any time T, we may take

$$\pi = 100° \, 31' \, 21.5'' + 61.70''(T - 1850.0) \qquad (7)$$

$$e = 0.0167711 - 0.000000424\,(T - 1850.0) \qquad (8)$$

whence, for the epoch 1887.0, $e = 0.01676$, and by putting A equal to unity, formula (6) gives

$$B = \sqrt{(1 - 0.0002809)} = 0.999860$$

The difference between A and B is far too small to be sensible upon any scale likely to be adopted for a model, and the earth's orbit may therefore be represented by a circle, in which the sun's distance from the center is e multiplied by the radius. For most models the diameter of this circle may conveniently be from two to six inches, and its circumference may be graduated to show the position of the earth at intervals of 5, 10, or 15 days. The circle having been cut out of stiff cardboard, one of its surfaces should be marked "North Side," and the other "South Side." When the former side is viewed the direction of the earth's motion is counter-clockwise; when the latter, clockwise. For effecting the graduation to show the position of the earth, Table I will be convenient.

The position of the comet in its orbit is defined by its true anomaly, which is the angle at the sun included between the comet and its perihelion. If for any comet we put

m = mean daily motion ;
M = mean anomaly at the time t ;
E = eccentric anomaly at the time t ;
v = true anomaly at the time t ;

then, in a parabolic orbit

$$m = \frac{188171''}{q^{\frac{3}{2}}} \qquad (9)$$

and the mean and true anomalies are connected by the equation

$$m(t - T) = M = 75 \tan \tfrac{1}{2}v + 25 \tan^3 \tfrac{1}{2}v \qquad (10)$$

where $(t - T)$ must be expressed in mean solar days. The labor of solving this equation is avoided by the use of Barker's table, some form of which is given in almost every treatise on the computation of orbits.

TABLE I.—LONGITUDE OF THE EARTH IN ITS ORBIT AT GREENWICH MEAN NOON.

NOTE.—This table is for the year 1886, being the second after bissextile; but for the purpose of model making, it will suffice for any year in the latter half of the nineteenth century.

Date.	Longitude.	Date.	Longitude.	Date.	Longitude.
January 1.....	101° 04′	May 1.....	221°	September 1...	338° 56′
" 11.....	111 16	" 11.....	230	" 11...	348 39
" 21.....	121 26	" 21.....	240 19	" 21...	358 25
February 1.....	132 37	June 1.....	250 52	October 1...	8 14
" 11.....	142 45	" 11.....	260 26	" 11...	18 06
" 21.....	152 50	" 21.....	269 58	" 21...	28 02
March 1.....	160 52	July 1...	279 31	November 1...	39 01
" 11.....	170 52	" 11...	289 03	" 11...	49 04
" 21.....	180 49	" 21...	298 35	" 21...	59 09
April 1.....	191 41	August 1...	309 06	December 1...	69 17
" 11.....	201 31	" 11...	318 42	" 11...	79 26
" 21.....	211 17	" 21...	328 19	" 21...	89 37

For an elliptic orbit whose semi-major axis is A

$$m = \frac{3548.2''}{A^{\frac{3}{2}}} \tag{11}$$

Periodic time in days $= \dfrac{360}{m} = 365.26\, A^{\frac{3}{2}}$ (12)

and the mean, eccentric, and true anomalies are connected by the equations

$$m\,(t - T) = M = E - e \sin E \tag{13}$$

$$\tan \tfrac{1}{2}\, v = \sqrt{\left[\frac{1+e}{1-e}\right]} \tan \tfrac{1}{2}\, E \tag{14}$$

Number (13) is the celebrated Kepler's equation. Owing to its transcendental form it cannot be solved directly, and notwithstanding all the labor and ingenuity which have been expended in attempts to discover an easier mode of solution, that by trial and error is still in general use.

Having now given all the definitions and formulæ necessary in constructing figures of the orbits of heavenly bodies, we have next to explain how these figures must be united in order that their mutual relations may be the same as those of the orbits they represent. To make that part of our subject perfectly clear we will describe in detail the construction of several models belonging respectively to different classes of comet orbits.

EXAMPLE I.—Let it be required to construct a model of the orbit of the comet, 1840, III, from the following parabolic elements:

$$T = 1840, \text{ April } 2d\ 12h, \text{ Greenwich Mean Time.}$$
$$\omega = 138°\ 16' \qquad i = 79°\ 51'$$
$$\Omega = 186\ \ 04 \qquad q = 0.7420$$

Motion direct.

In these elements the mean distance between the earth and the sun is taken as unity. In our model let us make that distance two inches. Then, to represent the earth's orbit we draw a circle of 2 inches radius upon a piece of cardboard about 0.029 of an inch thick. The distance of the sun's place from the center of this circle is equal to the radius of the circle multiplied by the value of e found from equation (8), namely,

$$2 \times 0.017 = 0.034 \text{ of an inch,}$$

and for the longitude of the perihelion, equation (7) gives 100° 11'. Accordingly, we draw the diameter AB, Figure 1, through the circle to represent the line of apsides; mark the point S upon it, at a distance of 0.034 of an inch from the center C, to represent the sun's place; and draw through S the line of equinoxes, DE, making an angle of 100° 11' with the line of apsides. The north side of the orbit being uppermost, the angle of 100° 11' must be laid off clockwise (in the direction of the motion of the hands of a clock) from A, the perihelion end of the line of apsides; and the number 100° 11' being affixed to the perihelion end of the line of apsides, the numbers 0° and 180° must be affixed to the two ends of the line of equinoxes in such a way that the numbers may increase counter-clockwise around the circle. These numbers represent longitudes in the earth's orbit, and the point marked 0° is called the vernal equinox because the *sun* passes through it in the spring. As the longitude of the earth is always 180° less than that of the sun, the earth does not reach the vernal equinox until about September 21. The circumference of the circle may now be graduated to show the place of the earth at every fifth day of the year, or less frequently if desired; but

in making this graduation, the center from which the angles are laid off must be the place of the sun, S, and not the center of the circle. Lastly, we set off the longitude of the comets $\Omega = 186° 04'$ counter-clockwise from the vernal equinox, and through the sun and the point thus found we draw FG, the line of intersection of the plane of the comet's orbit with that of the earth.

The next step is to lay down the curve destined to represent the comet's orbit, and for that purpose the coördinates of a sufficient number of points in the curve are required. To compute them we need an expression for the relation between y and x, and recalling that the q of our model is the q of the elements multiplied by the mean radius of the earth's orbit, namely, two inches, we have from equation (3)

$$y^2 = 4(2\times 0.742)x = 5.936x \qquad (15)$$

Then, having written down the values of x given in Table II, we find from (15) that when $x = 0.5$ the value of y^2 is 2.968 inches, and accordingly, that number is written opposite 0.5 in the table. As the interval between successive values of x is 0.5, for $x = 1.0$, $y^2 = 2.968 + 2.968 = 5.936$; for $x = 1.5$, $y^2 = 5.936 + 2.968 = 8.904$ inches, and so on; all the values of y^2 being formed by successive additions of 2.968, and the last one being verified by means of formula (15), to guard against possible errors of addition. The completion of the computation is then most conveniently effected by means of a table of square roots, from which the required values of y can be taken with the values of y^2 as arguments. Next, upon a piece of cardboard of the same thickness as that employed for the earth's orbit a straight line, HI, Figure 2, is drawn to represent the axis of the comet's orbit, and upon that line perpendiculars are erected at intervals of half an inch, as shown in the figure. Beginning at the left, these perpendiculars are marked successively 0.0, 0.5, 1.0, 1.5, etc., to correspond with the values of x in Table II; the ys belonging to these xs are set off upon the respective perpendiculars, both upward and downward, from the line HI; and the desired parabola is drawn

with a free hand through the points thus laid down. As explained in connection with equation (15), the perihelion distance is 1.484 inches, and the place of the sun S', is found by setting off that distance from the vertex of the curve upon the line HI. Lastly, with the sun as a center the angle $\omega = 138° 16'$, is set off clockwise from the perihelion, H, and through the sun and the point so found the line KL is drawn to define the intersection of the plane of the comet's orbit with the plane of the earth's orbit. That surface of the cardboard upon which we have been working must be marked "North Side" because it represents the northern side of the comet's orbit; and as the comet's motion is direct, it will move counter-clockwise along the parabola. A comet with retrograde motion would move clockwise. The instant of the comet's perihelion passage is known, because it is one of the elements of the orbit; but if we desire to mark the position of the comet for any other date, we must compute its true anomaly for that date and lay it off from the perihelion in the proper direction. A general explanation of the method of performing the computation has been already given, and it is needless to enter into further details here because there are several different forms of Barker's table and whatever form the reader may possess will be accompanied by all necessary explanations for its use.

Having thus laid down the orbits of the earth and comet,

TABLE II.

x	y^2	y	x	y^2	y
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
0.5	2.968	± 1.723	3.5	20.776	± 4.558
1.0	5.936	2.437	4.0	23.744	4.873
1.5	8.904	2.984	4.5	26.712	5.169
2.0	11.872	3.446	5.0	29.680	5.448
2.5	14.840	3.852	5.5	32.648	5.714
3.0	17.808	4.220	6.0	35.616	5.968

they must next be cut from the cardboard with the utmost care, the lines of the orbits being followed very exactly; and then, to permit the union of the two disks so obtained, a slit

of the same width as the thickness of the cardboard must be cut in each of them in the line of intersection of the orbits; that in the earth's orbit extending from the place of the sun, S, to F, the longitude of the $\Omega = 186° 04'$, and that in the comet's orbit extending from the place of the sun, S', to L, at longitude $(\omega + 180°) = 318° 16'$. Before cutting these slits the lines SM and $S'N$ should be drawn (the latter upon both sides of the cardboard) from the sun, at right angles to the intersection of the planes of the orbits, to serve as guides in putting the pieces together; and in cutting, care should be taken to hold the knife so that it makes the same angle with the cardboard as will be made by the other orbit when the two are united.

The slits having been cut, each orbit must be slipped into the other in such a way that the point F falls upon P, the point L near G, and the point S of the line SM upon S' of the line $S'N$, and then the orbits must be fixed together in that position by gumming a narrow strip of paper over the joint throughout its whole length. The strip of paper employed may be from one-quarter to three-eighths of an inch wide, and in order to make a good job, half its width should be gummed and applied to one of the orbits, and then the other half should be gummed and applied to the other orbit. The most convenient way of effecting this will be, 1st, to fold the paper lengthwise down its middle, thus reducing it to half its width; 2nd, to gum one side of the folded strip, and apply that side to one of the orbits before the two are united, taking special care that the fold of the paper lies accurately along the edge of the slit, and extends beyond it in the same straight line sufficiently far to cover the slit in the other orbit when the two are united; 3d, to gum the other side of the folded strip, taking care before doing so to insert a piece of waste paper within the fold in order to prevent any accidental smearing of the model; and lastly, to put the two orbits accurately together as described above, and then to unite them by smoothing the gummed paper down upon the one to which it has not hitherto been applied.

In order to complete the model it yet remains to fix the angle between the planes of the two orbits by inserting a triangular piece of cardboard, $S''M'N'$, Figure 3, of the same thickness as that used for the orbits themselves. The sides $S''M'$ and $S''N'$ of this triangle must have respectively the same lengths as the lines SM, Figure 1, and $S'N$, Figure 2, and the angle included between these sides must be equal to i, the inclination of the plane of the comet's orbit to the plane of the ecliptic, which in the present case is $79° 51'$. In the way described above, narrow strips of paper folded down the middle must be gummed to the edges $S''M'$ and $S''N'$ of the triangle, care being taken that the fold of the paper lies accurately along the edges in question; and then, after gumming the free sides of the strips, the triangle must be inserted between the orbits with its angle S'' at the place of the sun, and its sides $S''M'$ and $S''N'$ coinciding with the lines SM and $S'N$ on the orbits, and it must be fixed in that position by means of the gummed paper.

EXAMPLE II.—To illustrate the difference between the two systems of stating a comet's elements, let it be required to construct a model of the orbit of the comet 1874, VI, from the following parabolic elements, which are given both according to the old system and according to Gauss' system:

Old System.

T = 1874, Oct. 18*d* 23*h*, Paris Mean Time.

$\omega = 343° \ 43'$ $\qquad i = 80° \ 47'$

$\Omega = 281 \ \ 58$ $\qquad q = 0.5083$

Motion retrograde.

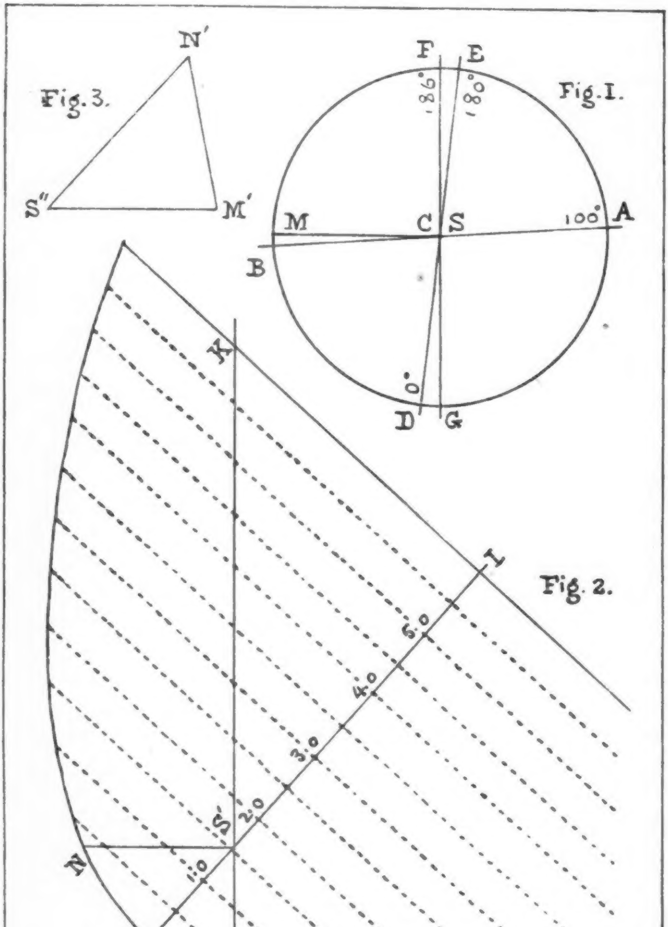
Gauss' System.

T = 1874, Oct. 18*d* 23*h*, Paris Mean Time.

$\omega = 16° \ 17'$ $\qquad i = 99° \ 13'$

$\Omega = 281 \ \ 58$ $\qquad q = 0.5083$

The orbit of the earth having been drawn, and the place of the sun together with the line of equinoxes having been marked; the line of nodes, or in other words, the line of intersection of the planes of the earth's and comet's orbits, must be


N′
Fig. 3.
S″
M′
F
E
186°
180°
Fig. I.
M
C
S
100°
A
B
0°
D
G
K
I
Fig. 2.
6.0
4.0
3.0
2.0
S′
N
1.0
H
L

found by setting off the longitude of the comet's ☊ = 281° 58′ counter-clockwise from the vernal equinox; the direction of the earth's motion must be marked as counter-clockwise; and that side of the cardboard upon which the drawing is made must be marked "North Side"—all these operations being performed precisely as in Example I. Next, the orbit of the comet together with its axis must be drawn, the place of the sun must be marked, and the line of nodes must be found by setting off the angle ω clockwise from the perihelion. After that the mode of procedure will depend upon the system of elements employed. If the old system is used the direction of the comet's motion must be marked as clockwise, because it is retrograde, and the side of the paper upon which the drawing is made must be marked "North Side;" but on the contrary, if Gauss' system of elements is used the direction of the comet's motion must be marked as counter-clockwise, and the side of the paper upon which the drawing is made must be marked "South Side." In either case the two disks representing the orbits of the earth and comet must be cut out and put together in the way described in Example I, care being taken that the "North Sides" of the disks face in the same direction, and of course the finished model will be the same whichever system of elements may have been employed.

A most instructive experiment can be made as follows: Let the old system of elements be used to lay down the orbit of the comet, its line of nodes, and the direction of the comet's motion, upon one side of a piece of thin paper; and then let the paper be turned over, and let the lines drawn upon its face be traced upon its back by looking through it. As the front of the paper represents the north side of the orbit, the back necessarily represents the south side, and the lines traced upon the latter will be found to be precisely those required by the Gaussian system of elements. Thus it will be impressed upon the memory that in the case of a comet having retrograde motion the old system of elements represents the orbit as seen from the north side of the ecliptic, while the Gaussian system represents it as seen from the south side of the ecliptic.

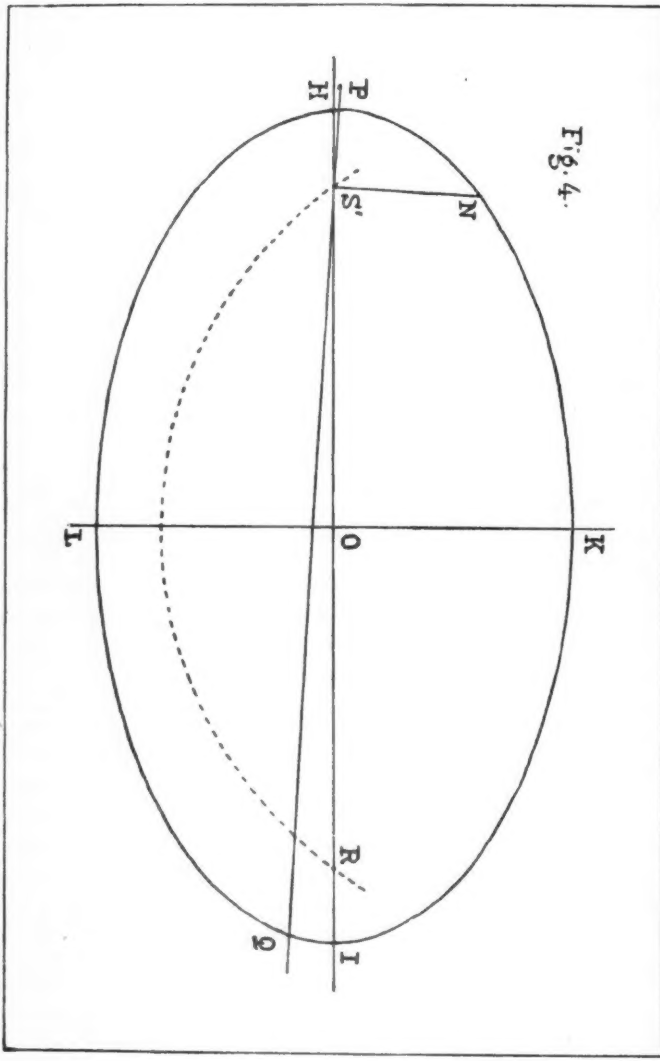


Fig. 4.

EXAMPLE III.—Let it be required to construct a model of the orbit of Encke's comet from the following elliptic elements:

T = 1871, Dec 28d 18h, Greenwich Mean Time.

$\pi = 158^\circ\ 12'$	$q = 0.3329$
$\Omega = 334\ \ 34$	$e = 0.84936$
$i = \ \ 13\ \ 08$	Motion direct

With these elements we form the quantities

$e^2 = 0.7214$	$1 - e^2 = 0.2786$
$1 - e = 0.1506$	$\sqrt{(1 - e^2)} = 0.5278$

and then, by means of formulæ (5) and (6), we compute the semi-major and semi-minor axes, thus:

$$A = \frac{0.3329}{0.1506} = 2.210$$

$$B = 2.210 \times 0.5278 = 1.167$$

These results are in terms of the mean distance between the earth and sun as unity, and they show that if that distance is made two inches, the length of the major axis of the comet's orbit will be 8.840 inches, which is a convenient size for our model, and will therefore be adopted.

The construction of the circular cardboard disk of two inches radius to represent the earth's orbit will be precisely as in Figure 1, except that in accordance with formula (7) the angle between the line of apsides and the line of equinoxes will be 100° 43′ instead of 100° 11′, and the longitude of the ☊ will be 334° 34′ instead of 186° 04′.

To construct the disk destined to represent the comet's orbit, select a piece of cardboard of suitable thickness and upon it draw the straight lines HI and KL, Figure 4, intersecting each other at right angles. From their intersection O, set off OK and OL, each equal to the semi-minor axis B, whose length is $2 \times 1.167 = 2.33$ inches. With K as a center and the semi-major axis $A = 2 \times 2.210 = 4.42$ inches as a radius, describe an arc cutting HI in R and S', and these points will be the foci of the ellipse. Stick a pin in each of them. Tie a

small loop in a piece of thread; pass the loop over one of the pins, and take a turn around the other with the free end of the thread. The thread will then be fastened to the two pins, and will lie upon the paper stretched between them. Press the point of a pencil sidewise against the thread, and by gently slackening the free end of the latter, let the turn slip until when the pencil is held perpendicularly its point just reaches the end of the minor axis at K or L, the thread being at the same time quite tight. Then, a finger having been placed upon the free end of the thread to prevent it from slackening further, the ellipse required to represent the comet's orbit may be described by moving the pencil sidewise while its point is kept firmly pressed against the thread.

As either of the two foci may be taken to represent the place of the sun, let S' be selected, and with it as a center set off the angle $\omega = \pi - \Omega = 183° \ 38'$ clockwise from the perihelion, H, and through the sun and the point so found draw the line PQ to define the intersection of the plane of the comet's orbit with the plane of the earth's orbit. That surface of the cardboard upon which we have been working must be marked "North Side;" and as the motion of the comet is direct, it will move counter-clockwise along the ellipse. The orbit thus laid down must next be cut from the cardboard; a slit must be made in it from S' to P; a triangle with an angle corresponding to $i = 13° \ 08'$ must be designed in the same way as Figure 3; and finally, all the parts of the model must be united precisely as described in connection with Example I.

EXAMPLE IV.—In dealing with comet orbits it is frequently desirable to make a rough model as rapidly as possible, merely to obtain a clear idea of what the elements mean. For that purpose the scale of the model should be such as to give the earth's orbit a radius of about an inch; the paper or cardboard employed may be 0.010 of an inch thick; and there will be needed a set of four or five pattern parabolas, made of cardboard, vulcanite, or sheet brass, in which the distances from the focus to the vertex of the curve are respectively ¼, ½, 1, 1½, and perhaps 2, inches. By their aid the disks represent-

ing the orbits of the earth and comet can be made and put together (without the strips of gummed paper) in about ten minutes. As an example of such work let it be required to construct a rough model of comet *f* 1887 (Brooks) from the following parabolic elements:

T = 1887, Oct. 6.480, Greenwich Mean Time.

$\omega = 63° \ 18'$ $\qquad i = 44° \ 10'$

$\Omega = 84 \quad 33$ $\qquad q = 1.2223$

In our previous examples we have adopted a definitive diameter for the earth's orbit and have then constructed the parabola required to represent the comet's orbit; but in order to save time, we must now adopt one of our pattern parabolas to represent the comet's orbit and determine the radius of the corresponding circle required to represent the earth's orbit. As the perihelion distance of the comet is 1.222, and we wish the radius of the earth's orbit to be about an inch, it is evident that we may use either of the two pattern parabolas whose focal distances are respectively 1 and 1½ inches. To find the corresponding radius of the earth's orbit, the focal distance of the parabola must be divided by q, and thus we obtain from the first parabola $1 \div 1.222 = 0.818$ of an inch, and from the second $1.5 \div 1.222 = 1.227$ inches. The latter value seems preferable, and we adopt it. Accordingly, the 1½ inch pattern parabola is laid upon a suitable piece of paper or cardboard, and the comet's orbit is drawn by passing a pencil around the pattern; care being taken at the same time to mark the focus and one other point in the axis of the curve. The pattern is then removed; the axis of the parabola is drawn through the points marked; the angle $\omega = 63° \ 18'$ is set off clockwise from the perihelion end of the axis, and the line defining the intersection of the plane of the comet's orbit with the plane of the earth's orbit is laid down. Next, a circle of 1.23 inches radius is described to represent the earth's orbit, and a diameter is traced through it to indicate the line of equinoxes; the slight displacement of the sun from the center of the circle being quite negligible. From the vernal equinox

the longitude of the ☊ = 84° 33′ is set off counter-clockwise, and the line of intersection of the plane of the earth's orbit with the plane of the comet's orbit is plotted. Finally, the two orbits thus laid down are cut from the cardboard; the necessary slits are made along the line of their mutual intersection; a triangle with $i = 44° 10'$ is prepared; and the three pieces are put together, either with or without the application of strips of gummed paper. It may be well to add that the above elements show the comet f 1887 to be Olber's comet.

Summary.—The foregoing rules may be summarized in the following form which applies both to the old, and to Gauss' system of elements:

In laying down the earth's orbit—

A. The longitude of the ☊ must be set off counter-clockwise from the vernal equinox.

B. The direction of the earth's motion must be marked as counter-clockwise.

C. That side of the paper or cardboard upon which the drawing is made must be marked "North Side."

In laying down the comet's orbit—

A. The angle ω must be set off clockwise from the comet's perihelion.

B. The direction of the comet's motion must be marked as counter-clockwise; except when the motion is retrograde with i less than 90°, and then it must be marked as clockwise.

C. That side of the paper or cardboard upon which the drawing is made must be marked "North Side"; except when i is greater than 90°, and then it must be marked "South Side."

Caution.—After the parts of a model have been correctly drawn there are several ways in which they may be wrongly put together, and sometimes one or other of the slits in the disks representing the orbits require to be cut in longitudes 180° different from those stated in our examples. The following conditions are imperative, and must be satisfied by every model, namely:

1. Those surfaces of the two orbits which are marked "North Side" must face in the same direction.

2. The comet must pass *at its ascending node* from the southern to the northern side of the plane of the earth's orbit.

3. The comet's perihelion must lie on the northern or southern side of the plane of the earth's orbit according as $\omega = \pi - \Omega$ is less or greater than 180°.

To prevent mistakes, these conditions should be borne in mind when cutting the slits, and until a careful inspection has shown that they are satisfied, the strips of gummed paper should not be applied.

When i is very small it is difficult to cut the slits in the orbits and insert the cardboard triangle in the way described above, and even if that were successfully done, the larger orbit would conceal much of the smaller. In such cases it is preferable to cut an aperture in the larger orbit equal to half the diameter of the smaller, and to fasten the two together in the proper relative positions by glueing or screwing them to a wooden wedge placed between them; thus dispensing with the slits, and leaving the entire sweep of both orbits visible.

The elements from which a model is made should always be written upon the disk representing the comet's orbit after all the parts of the model are completed, and before they are permanently fastened together.

There yet remain two matters which require notice, namely, the distance to which the disk representing the comet's orbit should extend from the sun; and the method of finding the intersection of the plane of the comet's orbit with the orbits of planets other than the earth.

No comet has ever been seen at a distance from the sun so great as five times the mean radius of the earth's orbit, and they are seldom visible at more than two and a half or three times that radius. The latter limit is therefore sufficient for a model in all ordinary cases. When a periodic comet describes a very elongated ellipse it is both unnecessary and inconvenient to draw the entire curve by the method explained in connection with Example III, and in such cases it will usually be

preferable to lay down the part required by tracing it through points whose co-ordinates have been computed by means of formula (4).

In order to show how near a comet may approach to planets other than the earth, it is frequently desirable that the points at which the orbits of these planets intersect the plane of the comet's orbit should be marked upon the disk representing the latter. The readiest way of doing this is to determine for each planet the line in which the plane of its orbit intersects the plane of the comet's orbit, and then to mark the point of intersection of the planet's orbit with that line by setting off the proper distance from the sun. If we put η for the angle upon the plane of the comet's orbit between the intersections of that plane with the planes of the earth's orbit and the planet's orbit; i and i' respectively for the inclinations of the planes of the comet's and planet's orbits to the plane of the ecliptic, both recokoned from 0° to 180° in accordance with Gauss' system; and ☊ and ☊′ respectively for the longitudes of the ascending nodes of the comet and planet; then

$$\cot \eta = \frac{\sin i \cot i' - \cos i \cos (☊ - ☊')}{\sin (☊ - ☊')} \qquad (16)$$

or, by introducing the auxiliary θ,

$$\tan \theta = \tan i' \cos (☊ - ☊') \qquad (17)$$

$$\cot \eta = \frac{\sin (i - \theta) \cot (☊ - ☊')}{\sin \theta} \qquad (18)$$

The results derived from equations (16) and (18) are freed from ambiguity by the circumstance that η can never be in the second or third quadrant, and must therefore be taken in the first quadrant when $\cot \eta$ is positive, and in the fourth when $\cot \eta$ is negative. As η necessarily lies on the same side of the ecliptic as the planet to which it belongs, and is positive when north and negative when south of that plane, a further check is afforded by remembering that every planet is north of the ecliptic from the longitude of its own ☊ to ☊ + 180°, and south of the ecliptic from ☊ — 180° to ☊.

The planetary elements which may be required for use in equations (16) or (18) are given in Table III.

TABLE III.—PLANETARY ELEMENTS FOR 1850.

Planet.	i	☊
	° ′ ″	° ′ ″
Mercury	7 00 07.71	46 33 08.6
Venus	3 23 35.01	75 19 53.1
Mars	1 51 02.28	48 23 53.0
Jupiter	1 18 41.37	98 56 16.9

Planets having a less perihelion distance than the comet intersect the plane of the latter's orbit in two opposite points, one of which will in general be north, and the other south of the plane of the ecliptic.

By means of a carefully constructed model having the orbits of the earth and comet graduated to show the positions of these bodies at each instant of time, all problems relating either to the apparent position of the comet in the heavens, or to its position relatively to other bodies of the solar system, can be roughly solved with great rapidity. Hastily constructed models of the kind described in Example IV are of course less useful, but much valuable information may be obtained even from them. Perhaps it is scarcely necessary to add that the position of the comet in the heavens for any given date is taken from a model by laying a straight wire from the earth's place to the comet's place, and then reading off the latitude and longitude corresponding to the direction of the wire. The latitude is the angle of elevation of the wire above or below the plane of the ecliptic; and the longitude is the angle at the sun between the vernal equinox and a line drawn through the sun parallel to the projection of the wire upon the plane of the ecliptic.

Washington, D. C., October 31, 1887.

The phenomena of the planets will find place hereafter regularly in each issue of the MESSENGER.

THE PROGRESS OF ASTRONOMY DURING THE NINETEENTH CENTURY.

Looking back to the year 1800, we are astonished at the change. The comparatively simple science of the heavenly bodies known to our predecessors, almost perfect so far as it went, incurious of what lay beyond its grasp, has developed into a body of manifold powers and parts, each with its separate mode and means of growth, full of strong vitality, but animated by a restless and unsatisfied spirit, haunted by the sense of problems unsolved, and tormented by conscious impotence to sound the immensities it perpetually confronts.

Knowledge might then be said to be bounded by the solar system; but even the solar system presented itself under an aspect strangely different from that it now wears. It consisted of the sun, seven planets, and twice as many satellites, all circling harmoniously in obedience to an universal law, by the compensating action of which the indefinite stability of their mutual relations was secured. The occasional incursion of a comet, or the periodical presence of a single such wanderer chained by planetary or solar attraction to prevent escape to outer space availed nothing to impair the symmetry of the majestic spectacle.

Now, not alone have the ascertained limits of the system been widened by a thousand millions of miles, with the addition of one more giant planet and six satellites to the ancient classes of its members, but a complexity has been given to its constitution baffling description or thought. Two hundred and seventy circulating planetary bodies bridge the gap between Jupiter and Mars, the complete investigation of the movements of any one of which would overtask the energies of a lifetime. Meteorites, strangers apparently to the fundamental ordering of the solar household, swarm, nevertheless, by millions in every cranny of its space, returning at regular intervals like the comets so singularly associated with them, or sweeping across it with hyperbolic velocities, brought perhaps from some distant star. And each of these cosmical grains of dust has a theory far more complex than that of

Jupiter; it bears within it the secret of its origin, and fulfils a function in the universe. The sun itself is no longer a semi-fabulous, fire-girt globe, but the vast scene of the play of forces as yet imperfectly known to us, offering a boundless field for the most arduous and inspiring researches. Amongst the planets, the widest variety in physical habitudes is seen to prevail, and each is recognized as a world apart, inviting inquiries which, to be effective, must necessarily be special and detailed. Even our own moon threatens to break loose from the trammels of calculation, and commits "errors" which sap the very foundations of the lunar theory, and suggest the formidable necessity for its revision. Nay, the steadfast earth has forfeited the implicit confidence placed in it as a timekeeper, and questions relating to the stability of the earth's axis, and the constancy of the earth's rate of rotation, are amongst those which it behooves the future to answer. Everywhere there is multiformity and change, stimulating a curiosity which the rapid development of methods of research offers the possibility of at least partially gratifying.

Outside the solar system, the problems which demand a practical solution are all but infinite in number and extent. And these have all arisen and crowded upon our thoughts within less than a hundred years. For sidereal science became a recognized branch of astronomy only through Herschel's discovery of the revolutions of double stars in 1802. Yet already it may be, and has been called, "the astronomy of the future." So rapidly has the development of a keen and universal interest attended and stimulated the growth of power to investigate this sublime subject. What has been done is little—is scarcely a beginning; yet it is much in comparison with the total blank of a century past. And our knowledge will, we are easily persuaded, appear in turn the merest ignorance to those who come after us. Yet it is not to be despised, since by it we reach up groping fingers to touch the hem of the garment of the Most High.

Our next volume will have new dress and new cover.

A NEW CATALOGUE OF STARS.

The principal work of astronomers, past and present, has been to determine as accurately as possible, with the means which each age has afforded, the position of the stars in the heavens. From comparisons made of the observations at different epochs is determined what is called the proper motion of the fixed stars, and also the movement of our solar system through space. The first observations record roughly the stars most prominent to the eye; and from this beginning of a thousand stars or more, with the invention of astronomical instruments, the number has been extended into the hundreds of thousands. And it is on the positions of these catalogue stars that the determination of all bodies in the solar system depends. Hence the nicest accuracy has been sought for these stars of reference, and redeterminations are constantly being made for various investigations. The verified positions are published for the use of computers of orbits in various astronomical journals and publications.

Dr. C. H. F. Peters, director of the Litchfield observatory at Hamilton College, is one of the most persistent and painstaking observers of the present century, and his observations are universally recognized as of the greatest accuracy. His observations, which are in course of preparation for publication and which will fill many volumes, all depend on stars of reference, a large number of which he has determined himself.

Evidently one of the most valuable aids an observer can have is an accurate catalogue of the reference, or comparison, stars, with their position at a given period. Such an aid, a boon to astronomers, has been in process of preparation for several years at the Litchfield observatory. Under Dr. Peters' direction his able assistant, Prof. Charles A. Borst, has reduced these stars to an epoch and constructed them into a catalogue, which will be an inestimable benefit to observers and an enduring monument to his own industry and attainments. In the prosecution of his work Professor Borst has gathered the stars from the various astronomical publications for the last half century, and made the computations for the reduction to

the epoch of the catalogue from the years in which they were observed. This part of his work was greatly facilitated by access to Dr. Peters' library, which is said to be the most comprehensive and complete in astronomical literature.

Thus has been constructed a catalogue of 30,000 stars, which will be of inestimable utility to astronomers who observe, and a most valuable acquisition to science. Little can be guessed by one not familiar with such work, of the honest work and painstaking care which this catalogue will represent. Thirty thousand stars, each computed to the present epoch, and each computation verified with the greatest care and accuracy! The computations fill several thousand folios, and have occupied Professor Borst's time during the past six years. The work is now virtually ready for the printer, and it is expected that its publication will be achieved the coming winter.—*Utica Herald, Nov. 5, 1887.*

PHOTOMETRIC OBSERVATIONS OF ASTEROIDS.

HENRY M. PARKHURST.

The variations in the brightness of the stars, are irregular, and affect so large a proportion of the stars, that uniformity of standard can only be secured by employing the means of large numbers. In the Harvard Photometry, Polaris is assumed to be invariable, and is made the standard. Analysis of more than 2000 observations, which would betray variability by causing the other stars to appear to vary simultaneously, proves that there is no change as yet appreciable. It is safe to continue to rely upon it, only because it is so continuously employed that any change which may occur, can be at any time ascertained with precision.

It was hardly to be expected that a star, in the process of combustion, should remain of unchanged brightness. It may be either the flickering light of burning gas, or the gradually waning light of an incandescent body. In the planets, shining by reflected light only, we have a standard as invariable as the illumination of our sun, but subject to certain periodic changes, the extent of which may be ascertained. Of the

planets, the asteroids are especially useful as standards of comparison for telescopic stars.

My own photometric observations of the asteroids commenced in April last, when there were four grouped together in the neighborhood of Regulus, and easily identified upon the ecliptic charts, and which could be frequently all observed in one series. I have since had no opportunity to compare asteroids with each other, but have compared several with comparison stars, with the standard of the Harvard Photometry.

In my first reduction I assumed invariable brightness, when reduced to the distances unity. The correction for illuminated surface, amounting to a few hundredths of a magnitude, I soon found to be inappreciable in comparison with much greater changes depending upon phase. This change I have found to vary, with different asteroids, ranging from M .13 for each degree of change in the angle P, the angle at the asteroid, to about M .01. My results, needing correction after the comparison stars have been better determined, have been, M .130, M .116, M .012, M .013, M .007.

The only case in which I have found marked evidence of change probably from rotation, is in the observation of Harmonia. On six evenings the results agreed within a few hundredths of a magnitude; on the other three evenings, the discrepancy was .70, .38, .74, brighter. I am confident that there was no error from misidentification. It is possible that light clouds partially obscured my comparison stars, but at present this seems incredible.

There is one series, which although unfinished, I wish to refer to specially, because it is supplementary to similar observations by Professors Pickering and Harrington; my observations of Vesta. Prof. Pickering in 1880 obtained for the brightness, reduced to the distances unity, 3 95. In the following year he made it 3.91; the mean being 3 93. Six years later, comparing with entirely different stars, in a different part of the heavens, and having no dependence whatever upon his previous observations, I brought out in my preliminary reduction, precisely the same value, 3.93. I subsequently found

Prof. Harrington's observations in 1883, also entirely independent, which gave the value, 4.10, or M .17 greater. These observations had not been corrected for phase.

The agreement between these results proves that our sun has not within the last seven years, varied appreciably in brightness; it proves that whatever irregularities there may be from rotation or other unknown causes, the light of the planet Vesta, in a series of observations, is a reliable standard of comparison; and I think it also proves that it is not yet quite time to "call a halt."

The observations were continued up to Sept. 19. Applying the correction for phase deduced from the observations, M .02 for each degree of the angle at the asteroid, to each of the four series of observations, the brightness, reduced to distances unity and corrected for phase, was as follows:

Pickering,	1880	M 3.59
"	1881	M 3.54
Harrington,	1883	M 3.65
Parkhurst,	1887	M 3.45
Final mean,		3.52

In the final mean, I have given weights to the several series according to the number of comparion stars employed, and independently compared with Polaris. The weight to be allowed for the meridian photometer observations of Pickering was somewhat arbitrary, but its amount does not perceptibly affect the result.

The mean error of my observations did not appreciably exceed the mean errors in observing the comparison stars.

EDITORIAL NOTES.

This number completes Volume VI of the MESSENGER, and hence nearly all subscriptions for the new volume of 1888 are due. If payment be made in advance, or before Jan. 20, the usual price of two dollars will be charged; if made later, $2.50. Subscribers are respectfully asked to notify us promptly if continuance of the MESSENGER is desired.

The Representation of Comet Orbits by Models is the title of an instructive and very carefully written article by Professor William Harkness of the U. S. Naval Observatory. Though unusual space is given to this theme, the details of making models of comet orbits are so fully and plainly stated, that even students of elementary astronomy could do the work well if so inclined. We think it not too much to say that every teacher of astronomy, in any grade of school, would find these home-made models very useful aids in conveying to their students definite ideas of the motions of comets and planets.

Recent Showers of Meteors (continued from Sidereal Messenger, Sept. and Oct., 1887, p. 288).—During the three months from August 1 to October 27, 1887, I spent 127½ hours in observation and saw 1144 meteors. On the whole the weather, though not exceptionally good, has proved tolerably favorable for this branch of work. From the many radiant points determined I have selected the following as representing the best streams recently seen here:

Epoch of Shower.	Night of Max.	Radiant Point. R. A. Dec.	No. of Meteors.	Appearance.
August 6-25	Aug. 25	334° + 58°	10	Rather swift.
August 7-22	Aug. 21	73 + 41	10	Very swift; streaks.
August 7-23	Aug. 23	327 + 48	8	Slow; faint.
August 10-24	Aug. 24	135 + 78	7	Swift.
August 10-24	Aug. 24	349 + 49	11	Rather slow.
August 14-23	Aug. 23	264 + 62	7	Slow; trained; brilliant.
August 20-24	Aug. 21	54 + 71½	10	Very swift; streaks.
August 20-24	Aug. 21	347 + 15	7	Slowish.
August 20-25	Aug. 23	43 + 39	9	Swift; streaks.
September 7-24	Sept. 19	5 + 10	15	Slow; short.
September 7-24	Sept. 17	64 + 22	8	Very swift; streaks.
September 7-24	Sept. 8	358 + 60	10	Slowish.
September 12-22	Sept. 18	28 + 72	7	Slowish; short.
September 13-24	Sept. 24	7 + 44	7	Slowish.
September 17-19	Sept. 18	41 + 38	7	Swift; streaks.
September 17-22	Sept. 22	335 + 58	12	Slow; bright.
October 11-14	Oct. 13	192 + 83	10	Slow; bright; trained.
October 11-15	Oct. 14	40 + 29	12	Swift; short.
October 11-21	Oct. 11	29 + 72	13	Swift; small; short.
October 11-24	Oct. 14	40 + 20	45	Rather swift.
October 12-21	Oct. 13	312 + 77	8	Swift.
October 14-15	Oct. 15	25 + 44	10	Slow; small; short.
October 14-20	Oct. 14	117 + 47½	8	Very swift; streaks.
October 14-21	Oct. 21	54 + 71	12	Swift.
October 14-21	Oct. 14	105 + 22	12	Very swift; streaks.
October 14-24	Oct. 14	135 + 68	11	Swift.
October 17-21	Oct. 20	47 + 44	8	Swift.
October 20-21	Oct. 21	125 + 43	7	Very swift; streaks.

In addition to these I re-observed those well known showers

the Perseids and Orionids. I have already given a table of my results for the Perseids up to the end of July (see MESSENGER, Sept. and Oct., 1887, p. 288) and now subjoin a list of the radiants for this stream determined here in August.

Date.	Radiant.	No. of Meteors.	Appearance.
1887.	R. A. Dec.		
August 6	42° + 55°	5	The Perseids
August 7	43 + 56	5	are swift, bright
August 8	43 + 56	6	meteors, leav-
August 10	42½ + 57½	22	ing phosphor-
August 11	45 + 57½	16	escent streaks.
August 14	53 + 57	8	

The remarkable displacement to the east as shown by the radiant this year fully confirms my previous observations reported in the SIDEREAL MESSENGER, April, 1886, p. 107.

I watched the shower of Orionids very closely in October, with an endeavor to trace any change also affecting its radiant but the displacement, if any, is too slight for determination. My observations were as follows:

Date.	Radiant.	No. of Meteors.	Appearance.
1887.	R. A. Dec.		
October 11-14	91° + 17°	5	The Orionids
October 15	91 + 16	17	are very simi-
October 17	90 + 15	3	lar, in their vis-
October 19	90½ + 15½	10	ible aspect, to
October 20	90 + 14½	22	the Perseids.
October 21	92 + 14	23	
October 24	91 + 16	9	

Allowing for the unavoidable errors of observation the radiating center of the shower seemed slackening at 91°+ 15°.

In addition to this annually recurring stream, we have this year been favored with a fine display of meteors from a radiant at 40°+ 20° near ε Arietis. I recorded 45 of its meteors in October and have previously referred to this system as a very prominent one at this epoch. (See *Monthly Notices*, Vol. XLIV, p. 24-26.) W. F. DENNING.

Bristol, England, Oct. 27, 1887.

The Supposed Satellite of Venus.—A very interesting and valuable paper entitled "Etude sur le Satellite Enigmatigue

de Venus," comes to us from the Royal Academy of Belgium, through the courtesy of the author, Mr. Paul Stroobant. It is well known that in a certain number of instances, all previous to the present century, observers have seen in the same field of the telescope with Venus a small object which might have been recognized as a satellite, but for the fact that it was seen only at intervals, sometimes of fifteen or even fifty years, and that it has not been seen at all during the present century, although we possess telescopes of far greater power and better quality.

The author says that his attention was called to the study of this question by an article published by Professor Houzeau in 1884, in which it was suggested that the observations might be explained by supposing the existence of a small planet revolving in an orbit a little exterior to that of Venus. Mr. Stroobant rejects this hypothesis, as well as the seven other hypotheses which have been offered at various times, and leads us to the surprising conclusion that in nearly every instance the observers saw nothing but fixed stars which can be identified upon the star charts which we now possess. In several cases it seems astonishing that the observers did not satisfy themselves that they had not observed a fixed star. For instance, in the observation of Roedkioer and Boserup at Copenhagen, August 4, 1761, the observers took χ_4 Orionis (5th mag.) for the satellite, while using χ_3 Orionis (5½ mag.) as a comparison star. Again, the stars observed for the satellite by Roedkioer, July 18, August 7, and August 11, 1761, were *m* Tauri, (5th mag.), 71 Orionis (6th mag.), and ν Geminorum (4½ mag.), and that by Horrebow, Jan. 3, 1768, θ Libræ (4½ mag.). The motion which the last observer ascribed to the satellite during his observation is exactly equal and in the contrary direction to that of Venus.

The author gives first in tabular form a summary of all the observations of the supposed satellite, 33 in number, from 1645 to 1768, then the data for computing the apparent places of Venus upon the celestial sphere at the moments of observation. He then reviews the various hypotheses which have

been advanced in regard to these observations and finally compares the computed places of the satellite with the star charts. In a few instances the identification of the object observed with an existing star is not quite satisfactory, and it may be possible that some one of the brighter minor planets has thus been observed. In one case, Roedkioer, March 4, 1764, it seems very probable that the planet Uranus was observed, the distance between the two planets being then only 16′.

In an appendix the author gives the original text in regard to each of the observations. This appendix alone makes the paper very valuable to astronomers to whom the original texts are difficult of access. There are added three plates giving star charts upon which the positions of Venus and the observed satellite are noted. H. C. W.

N. C. Dunér, Astronomer at Lund Observatory, Sweden, writes Mr. J. A. Brashear recently, a very complimentary letter concerning the Rowland gratings which were supplied to him by Mr. Brashear. Mr. Dunér finds the optical quality of the gratings "most satisfactory," and he says "the brightness of the spectra of the 3d, 4th and 5th orders are really surprising, and in fact greater than that of the 2d, a circumstance which is very favorable for my researches in which great dispersion is desirable." Other foreign observers seeing Mr. Dunér's grating have asked for copies of some as large as the 6-inch concave grating.

Comet Meteor Radiants.—Below will be found the radiants and distances of the new comets of last year. The present list is a continuation of my last, given in SIDEREAL MESSENGER, 1886, p. 152. The first column contains the current number; the second gives the designation of the comet in the order of perihelion passage of the year; the third, the discoverer; the fourth, the day of the earth's passage through plane of comet's orbit; the fifth designates the nearest node; the sixth, the distance of this node from the earth's orbit in the order $R - r$, the earth's distance from the sun being taken as

unity; the seventh and last column gives the radiant for the day in question.

Current Number.	Designation of Comet.	Discoverer	Earth's Passage through Node.	Designation of Node.	Earth's Distance from Node.	Radiant.	
						R. A.	Dec.
1............	1886 I........	Fabry	April 27....	Descending..	+ 0.11	322.6°	+ 36.9
2............	1886 II.......	Barnard ...	May 30.....	Descending..	+ 0.36	350.8	+ 49.0
3............	1886 III......	Brooks	July 11.....	Ascending...	+ 0.05	20.5	— 40.9
4............	1886 IV	Brooks	May 13.....	Descending..	— 0.36	158.2	+ 54.2
5............	1886 V	Brooks	Oct. 6......	Descending..	+ 0.72	88.0	+ 78.5
6............	1886 VII	Finlay	Nov, 16	Ascending...	— 0.14	263.3	— 36.4
7............	1886 VIII	Barnard ...	June 9.....	Ascending...	— 0.56	3.7	— 61.6
8............	1886 IX	Barnard ...	Feb. 6......	Ascending...	— 0.25	173.5	— 32.7

Harvard College Obs'y, Oct. 22, 1887. O. C. WENDELL.

The Star of Bethlehem.—In your issue of Sept.-Oct. you have an article on the above subject. May I ask your permission to call attention to one point which seems to me to be of vital importance, and which appears to be entirely overlooked? Apart from any question as to the actual occurrence, a scientific consideration of the various hypotheses put forward to account for it would, in the first place, divide these hypotheses into two groups—terrestrial and celestial. If this be done, and we then proceed to consider the Bible account,—that the star "went before them till it came and stood over the place where the young child was,"—we shall see at once that none of the celestial phenomena put forward to account for the story are admissible. The "planets" are "wanderers," to be sure; but the perturbations which led to the discovery of Neptune would be nothing to what would have occurred if Jupiter and Saturn or Jupiter and Mars had so far wandered from their ordinary courses as to lead the Magi to imagine that they were following an actually moving body preceding them in their travels. And when the "star" had arrived at its destination, and "stood over" the place—what then? Suppose Jupiter to be in the zenith, what would the editor give for his chance of finding some buccaneers' buried treasure, because he knew the planet "was over the place" where the treasure was? Of the four heads under which the subject was considered in the article, three, I think, must be ruled out at once,—those relating

to planets, comets and stars,—and the first left in possession of the field. For, not only does this give an ample explanation of the matter, but all the other suppositions are tainted with the objection that they are extensive interferences with the natural order which do not evade the imputation of miraculousness, which is the only objection to the first head; or that they are merely accidental coincidences, in which case the Divine guidance would appear to be entirely lost sight of. I think you will agree with me, that those who are not satisfied with the explanation under the first head, are not likely to be satisfied by any possible hypothesis under any of the others.

Toronto, Ont., Oct. 22, 1887. JAS. T. ELLIS.

Observing Nebulæ at Leander McCormick Observatory.—A method of obtaining the places of new nebulæ, lately introduced at the Leander McCormick Observatory, is given with the hope that its use will insure better places than have been previously, in many cases, given. The method has been only lately rendered convenient for use on southern nebulæ by the publication of the Southern Durchmusterung. The eye-piece on a filar micrometer is used in sweeping; the wires being in the meridian the differences of right ascension is observed, by means of a chronometer, or the beats of the armature of the driving clock, between the nebula and two stars of such magnitudes as to insure their being found in the Durchmusterung. The micrometer is then turned through 90°, the fixed wire placed on the nebula, the telescope is then moved until the star is in the field, and the movable wire placed on it. The reading of the micrometer together with the reading at the coincidence of the wires gives the difference of declination. The position of the telescope is obtained roughly, this with the observed difference of right ascension between the stars serves to identify them almost beyond a doubt. In this way in a few minutes the place of the nebula can be obtained with very little chance of error, and as accurately as is warranted by the star places in the Durchmusterung. If an electric light, the brightness of which is regulated by a switch close at hand,

is used to illuminate the wires almost any nebula visible in the telescope can be observed in the above way; and the accurate place of almost any visible nebula with a condensation can be obtained in the usual way with a filar micrometer with such illumination.

A more convenient way of obtaining the places of nebulæ approximately would be to have an eye-piece like that of a square bar micrometer with three of the bars removed and the fourth graduated to, say, ten, or twenty, seconds of arc. This one bar would not interfere much with sweeping, places could be obtained rapidly, and probably as accurately, in the case of diffuse nebulæ, as in any other way. FRANK MULLER.

The Scientific American, one of the best papers of its kind, is doing astronomy a real service in a popular way. The issue of Sept. 24 contained an instructive article, fully illustrated, on the theme "How Telescopes are Made," showing cuts of the Alvan Clark shops at Cambridgeport, Mass., inside and outside, and describing their methods of work intelligently and somewhat in detail. The group of three sitting at a table, representing the senior Alvan Clark (recently deceased), Alvan G. Clark and George Clark, is true to life.

In the number of October 15 we have another illustrated article entitled "Harvard Observatory and the Henry Draper Memorial." Those who have not visited these places will get definite knowledge of them by these articles.

Charles B. Hill, assistant at the Chabot Observatory, Oakland, California, finds opportunity to make observations such as the MESSENGER has given frequently, outside of the time devoted to public uses of the observatory for which it was built. The instruments are, an 8½-inch Clark equatorial, a 4-inch Fauth Transit and the necessary astronomical time pieces.

Mr. D. Appel, of Cleveland, O., recently returned from Europe. While in Germany he visited the principal cities of the Rhine, thence he made a tour through Italy and Switzerland

and back again to Germany, reaching Berlin Aug. 16. He was at Bramberg, Aug. 18, with a view of observing the corona during the total solar eclipse. At the same place were some of the Vienna astronomers, Dr. von Konkoly and von Gothard. Clouded skies defeated observations. Mr. Appel visited several of the leading observatories while abroad.

Report of the Leander McCormick Observatory by the Director, Professor Ormond Stone, for year ending June, 1887, though brief, has many points of interest. The discussion of the nebula of Orion, respecting change of brightness, is quite in accord with views recently published by Professor Holden, although the interesting details of structure and probable variation of relative brightness brought out by Professor Stone make the evidence fresh and quite conclusive. The examination and sketching of southern nebulae have been continued during the last year, resulting in 351 observations of miscellaneous nebulæ, and in the discovery of 270 nebulæ supposed to be new. Considerable work has also been done in the measurement of southern double-stars.

Catalogue of Red Stars.—A working catalogue of Red Stars, prepared by G. F. Chambers, of England, is a compilation of observations extending over a period of seventeen years from 1870. The telescope used in the earlier part of this work by Mr. Chambers, was a 4-inch refractor by Cooke, but all observations made after 1884 were by a 6-inch refractor by Grubb, having a low power eye-piece and a field of 1¼°. Star catalogues covering dates from 1804 to 1876 have contributed red stars to his catalogue, making a total number of 719. This is a very useful compilation in the furtherance of needful study of stellar physics.

The Asteroids.—An essay on the asteroids recently prepared by Professor Daniel Kirkwood, Bloomington, Indiana, and published by Messrs. J. B. Lippincott of Philadelphia, will be issued during the present month. Professor Kirkwood is authority concerning the astronomy of the minor planets.

Warner Observatory.—Dr. Lewis Swift has recently published a series of articles on popular astronomy in the Rochester "Union and Advertiser," entitled "Simple Lessons in Astronomy." They are profitable reading for any one, especially the student, the teacher and the amateur. In a recent letter he says that Dr. Dryer's new general catalogue of nebulæ will soon be ready for distribution. It contains 7840 nebulæ besides those of the postscript.

BOOK NOTICES.

Chauvenet's Treatise of Elementary Geometry, Revised and Abridged; by W. E. Byerly, Professor of Mathematics in Harvard University. Philadelphia: Messrs. J. B. Lippincott Company, Publishers, 1887, pp. 322, 8 vo., cloth, price $1.28.

Most students of mathematics known of the excellent text-books written by Chauvenet. As prepared by himself, we do not know of any better in the English language, generally speaking. Apparently this author could write on themes of pure mathematics, in close, exact and logical phrase, as a ready talker converses with a friend, or a genius speaks his living thoughts. His writings show instructive insight and naturalness. The only trouble that the preparatory schools have found with these books is that they were too comprehensive for common use. To meet this difficulty Professor Byerly has revised and abridged the Geometry, a copy of which has just reached us.

There are three things in this book that are mostly new and excellent. They are

(1) The introduction of *numerous* graded exercises in the body of the book, with suggestions to aid the student in solving the more difficult ones. (2) Stress laid on independent thinking on the part of the student as distinguished from mere memorizing; and (3) A syllabus of most important propositions and corollaries. The first point only needs to be stated; the second point is a wise step, urging teacher and student to adopt, at the earliest time possible, the true modern method in Geometry which is "demonstration at sight" as Latin and Greek are now taught to be read at sight in the best schools. The right use of this book must bring most desirable results.

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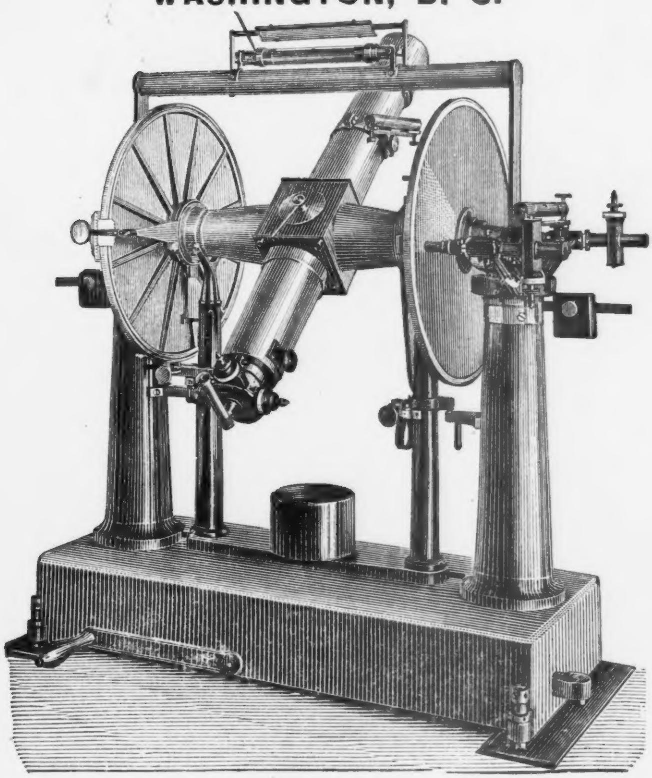
NOTE.—The numbers following the titles of articles are the pages of this volume. Those in parenthesis are the numbers of the Minor Planets.



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